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TRANSONIC PITCH DAMPING OF A DELTA WING AIRCRAFT DETERMINED PRO--ETC

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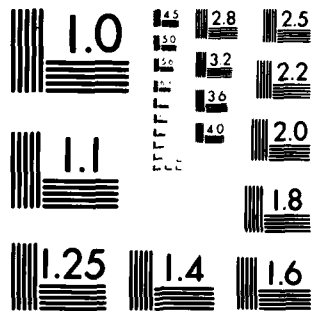
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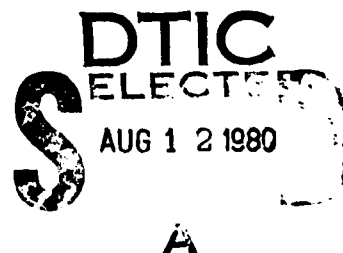
AERODYNAMICS REPORT 153

TRANSONIC PITCH DAMPING OF A DELTA WING
AIRCRAFT DETERMINED FROM FLIGHT
MEASUREMENTS

by

R. A. FEIK

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AERODYNAMICS REPORT 153

**TRANSONIC PITCH DAMPING OF A DELTA WING
AIRCRAFT DETERMINED FROM FLIGHT
MEASUREMENTS**

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(16) R. A. FEIK

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SUMMARY

A flight investigation aimed mainly at determining the pitch damping of a 60 degree Delta wing aircraft between $M = 0.7$ and $M = 1.2$ has been undertaken. Flight tests were conducted by ARDU and the measurements obtained have been successfully analysed at ARL using a modified Newton-Raphson parameter estimation procedure. The results for pitch damping as well as other longitudinal aerodynamic derivatives have been compared with wind tunnel and/or theoretical estimates. Further information has also been gained relating to the necessary conditions for successful extraction of parameters from flight data. The parameter estimation procedure has proved to be an effective analysis method even when only a modest amount of flight data, obtained using a relatively simple instrumentation package, has been available.

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16. ABSTRACT

A flight investigation aimed mainly at determining the pitch damping of a 60 degree delta wing aircraft between $M = 0.7$ and $M = 1.2$ has been undertaken. Flight tests were conducted by ARDU and the measurements obtained have been successfully analysed at ARL using a modified Newton-Raphson parameter estimation procedure. The results for pitch damping as well as other longitudinal aerodynamic derivatives have been compared with wind tunnel and/or theoretical estimates. Further information has also been gained relating to the necessary conditions for successful extraction of parameters from flight data. The parameter estimation procedure has proved to be an effective analysis method even when only a modest amount of flight data, obtained using a relatively simple instrumentation package, has been available.

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1. INTRODUCTION

In an earlier note¹ dealing with the analysis of longitudinal flight test data for a delta wing aircraft at Mach numbers of 0.71 and 0.96, a substantial discrepancy was identified in pitch damping at $M = 0.71$ between the value extracted from flight and that previously accepted. As the results at $M = 0.96$ were in reasonable agreement, the variation of pitch damping with Mach number over the transonic speed range was in question. The pitch damping derivative contributes both to the transient response of an aircraft to control input and also to the steady state manoeuvring performance such as a turn or pull-up, and is consequently an important part of any mathematical representation of an aircraft. Since it is difficult to measure in the wind tunnel or to calculate theoretically, especially at transonic speeds, estimates for use in a mathematical model need to be validated against flight data. In order to do this, a flight test program was initiated to measure the pitch damping over the Mach number range $M = 0.7$ to 1.2. The flight tests, including instrumentation and data recording, were conducted by the RAAF Aircraft Research and Development Unit (ARDU) at Salisbury, and the data obtained were analysed at ARL. The results of these tests, which produced data for other longitudinal derivatives in addition to pitch damping, are reported here.

The flight data were analysed using the modified Newton-Raphson parameter estimation procedure first documented in Reference 2. This is a response error method which compares model output with measured responses and systematically adjusts the model parameters in order to obtain a best fit. The parameters are assigned weighted *a priori* values, based, for example, on wind tunnel data, so that the estimated values of the parameters will tend towards their *a priori* values unless the flight data contains information requiring a different value for best fit. The method is now widely used in the analysis of aircraft flight dynamic tests and has been applied at ARL to both longitudinal¹ and lateral³ flight data including the analysis of fin loads.³ Reference 4 has shown that the method can also be used to obtain lift and drag characteristics from dynamic flight manoeuvres. A more detailed summary of the procedure as applied in the present investigation can be found in Reference 1.

Section 2 of this Note describes details of the mathematical model used in the analysis. The model includes provision for non-linear variation of pitching moment and lift curves with incidence. Section 3 deals with the test data details, including instrumentation and recording, and describes the pre-processing undertaken in preparation for the identification program. *A priori* values and weights used in the matching process are also summarised. The results are presented and discussed in Section 4 and comparisons made with wind tunnel and theoretical estimates where possible.

2. MATHEMATICAL MODEL

In this section a mathematical model is developed which describes the aircraft longitudinal short period response to an elevator input. A basic linear model is extended to include possible non-linearities with incidence in the pitching moment and lift curves. A further extension is required to deal with a time lag in the pitch rate measurements.

2.1 Basic Model

It is assumed that the aircraft is trimmed in steady level flight with speed V_0 , incidence α_0 and attitude θ_0 . Small disturbance motions about this initial state are produced by elevator inputs. The linearised small disturbance equations, in body axes, describing the short period response have been derived in detail in Reference 1. The model state equations take the basic form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (1)$$

and the calculated response is:

$$y = Fx + Gu + b \quad (2)$$

where

x is the vector of state variables,

u is the vector of control variables,

y is the response vector,

b is a measurement bias vector taken as zero initially,

A , B , F and G are matrices of parameters defining the model.

The parameter vector, c , contains some or all of the elements of A , B , F and G , the elements of b and the initial conditions.

2.2 Non-linear Aerodynamics

The incidence ranges covered at each Mach number in the present series of tests are summarised in Figure 1. Wind tunnel tests⁵ indicate significant non-linear variation of pitching moment with angle of incidence for the delta wing configuration under consideration, particularly at Mach numbers near 1. Following the approach successfully applied in Reference 1, the moment increment due to incidence is modelled by a cubic equation:

$$\Delta M = M_\alpha \alpha + M_{\alpha^2} \alpha^2 + M_{\alpha^3} \alpha^3 \quad (3)$$

To a lesser extent, the Z -force variation with incidence is also non-linear and a cubic variation is postulated as follows:

$$\Delta Z = Z_\alpha \alpha + Z_{\alpha^2} \alpha^2 + Z_{\alpha^3} \alpha^3 \quad (4)$$

In order to retain a linear mathematical model the α^2 and α^3 terms are treated as known inputs and become part of the input vector. Their values are either calculated directly from measured values of incidence, α_m or if measured values of incidence are not available, initial linear model calculations may be used to provide suitable values.

In addition to pitching moment and Z -force non-linearities it may be speculated, for example, that pitch damping may also be a function of incidence requiring an equation similar to Equations (3) and (4) above to model it. Each such extension further complicates the basic model and adds extra parameters to be identified. It is clearly desirable, from a computational point of view, to adopt as simple a model as possible so long as it adequately describes the physical system. The question of model structure determination is an important part of systems identification addressed by a number of authors.^{6,7,8} Methods have been developed to select a best model from a large number of candidate models. In the present case the number of options is relatively small and can be examined one by one. If a significant improvement is obtained in matching the test data then the extension to the basic model should be retained. Thus it was found that the modification described by Equation (3) led to significant improvements in the subsonic and lower transonic Mach number range and was thus included in the model. The modification described by Equation (4) was much less significant and it was subsequently decided to retain only the first two terms of Equation (4). On the other hand, modelling of pitch damping as a non-linear function of incidence produced little improvement and was thus excluded in the interests of model simplicity.

2.3 Pitch-rate Measurement Lag

Initial attempts to match normal accelerometer and pitch rate gyro measurements focused attention on an apparent lag of approximately 0.1 second in the pitch rate measurements when compared to initial mathematical model results. Further investigation pointed to the pitch rate gyro demodulator unit as being the source of this lag. To account for this the model was augmented by a first order lag equation as follows:

$$\frac{dq_m}{dt} + \frac{q_m}{\tau} = \frac{G}{\tau} q \quad (5)$$

where τ is the lag time constant and the gain term G is included to account for possible calibration and/or gyro misalignment errors in the pitch rate measurements, q_m . Both τ and G can be included in the parameter vector to be identified if required. Greatly improved results were obtained when the lag was accounted for by Equation (5) with τ initially set to 0.1 and G to 1.0.

The complete model used in the present investigation can now be summarised.

The state equations are:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{q}_m \end{bmatrix} = \begin{bmatrix} \frac{Z_a}{mV_e} & 1 & \frac{-g \sin \theta_e}{V_e} & 0 \\ \frac{M_a}{I_y} & \frac{M_q + M_{\dot{a}}}{I_y} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{G}{\tau} & 0 & \frac{-1}{\tau} \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \\ q_m \end{bmatrix} + \begin{bmatrix} \frac{Z_{\delta_e}}{mV_e} & \frac{Z_{\alpha^2}}{mV_e} & 0 & Z_0 \\ \frac{M_{\delta_e}}{I_y} & \frac{M_{\alpha^2}}{I_y} & \frac{M_{\alpha^3}}{I_y} & M_0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \alpha^2 \\ \alpha^3 \\ 1 \end{bmatrix} \quad (6)$$

The response equations are:

$$\begin{bmatrix} \alpha \\ q \\ \theta \\ q_m \\ \frac{gn_z}{V_e} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{Z_a}{mV_e} & \frac{Z_q}{mV_e} & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \\ q_m \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{Z_{\delta_e}}{mV_e} & \frac{Z_{\alpha^2}}{mV_e} & 0 & Z_0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \alpha^2 \\ \alpha^3 \\ 1 \end{bmatrix} \quad (7)$$

The parameter vector is:

$$c = [Z_a/mV_e, Z_q/mV_e, Z_{\delta_e}/mV_e, M_a/I_y, (M_q + M_{\dot{a}})/I_y, M_{\delta_e}/I_y, G/\tau, -1/\tau, Z_0, M_0, Z_{\alpha^2}/mV_e, M_{\alpha^2}/I_y, M_{\alpha^3}/I_y]^T \quad (8)$$

Z_0 and M_0 have been introduced to account for possible state bias. The Z_q term in Equation (7) was found to make very little contribution to the results and was subsequently set to zero in Equation (7) and omitted from the parameter vector, Equation (8). The measurement bias vector, b , has been assumed to be initially zero in Equation (7) but the biases in the measurements of q_m and gn_z/V_e were generally included among the parameters to be identified in Equation (8).

3. FLIGHT DATA

The form of the data and the test conditions are first described. This is followed by a discussion of the preliminary processing and corrections applied to the data. Finally, the parameter *a priori* values and weights are summarised.

3.1 Test Conditions

Tests were performed at the nominal test points summarised in Table 1.

TABLE 1
Nominal Test Points

Height (ft.)	Mach Number
30,000	0.7, 0.8, 0.9, 0.92, 0.94, 0.96, 0.98, 1.05, 1.1, 1.2
10,000	0.7, 0.8, 0.9, 0.92, 0.94, 0.96, 0.98, 1.05, 1.1, 1.2

The test flights, conducted by ARDU pilots, took place in clear atmospheric conditions with a minimum of turbulence. Two flights were made at 30,000 ft and one at 10,000 ft. In each case the set of Mach numbers was flown in the sequence shown in Table 1. The pilot was asked to trim the aircraft at each point and then, with pitch dampers off, apply a rapid stick doublet input sufficient to produce a transient normal acceleration increment of at least $\pm 1 g$. In some cases the pilot was able to apply a second doublet input after the response to the first had died down, at the same time checking that the Mach number remained as constant as possible.

At each test point the pilot recorded the values of Indicated Airspeed (IAS), Indicated Mach Number (IMN), height and fuel gone prior to initiating the trace recorder. The quantities recorded on the paper trace recorder included IAS, height, normal acceleration, pitch rate, port and starboard elevator positions and pitch damper position. Angle of incidence measurements were not available. The fast (about 4 inches/second) recording speed was used in most cases although some runs were recorded at a slower speed (about $1\frac{1}{2}$ inches/second). All instrumentation and data recording was undertaken by ARDU personnel. The paper trace records were then read by Mathematical Services Group at Defence Research Centre, Salisbury, and the final results, in Engineering units, provided to ARL in digitised form on magnetic tape. It was possible for DRCS to read the traces at the required sampling rate of 40 per second even at the lower trace recording speed.

The subsequent identification was based on matching the time histories of normal acceleration and pitch rate responses to the given elevator control input.

3.2 Pre-processing

True airspeed (V_T) was obtained using standard charts from IAS corrected for position error. The corresponding true Mach number, M_T , agreed with corrected IMN readings to within 0.01 for most cases. The variation of aircraft centre of gravity, mass and moment of inertia in pitch were calculated from the fuel gone readings using the mathematical model described in Reference 9. Using the same model, values of aircraft trim attitude, θ_e , and incidence, α_e , for each test point were established for subsequent use. In addition the following pre-conditioning was applied to the measured data:

- (a) Normal accelerometer readings were corrected for location error according to the equation:

$$n_z = n_{zm} + (x_a \dot{q} + z_a \dot{q}^2)/g \quad (9)$$

where n_{zm} is the measured acceleration, n_z is the value at the centre of gravity and x_a , z_a are the co-ordinates of the accelerometer relative to the aircraft body axes. Measurements of \dot{q} were not available so that the correction $x_a \dot{q}$ was only applied in the second stage of a two-part calculation procedure as described in Section 4.1.

- (b) Non-zero trim values of normal acceleration and port and starboard elevator positions were subtracted from the respective measured records so that the incremental values could be compared directly with the small disturbance model calculations. In addition the resulting port and starboard elevator settings were averaged to provide the required control input.

3.3 A Priori Values and Weights

The *a priori* values assigned to the aerodynamic derivatives in the mathematical model were mainly based on wind tunnel measurements (e.g. Ref. 5) or else estimated from data sheet sources. The non-dimensional values of the derivatives used are summarised in Table 2.

TABLE 2
A Priori Aerodynamic Derivative Values and Weights

M	$-C_{z_{\alpha}}$ per degree	$-C_{z_{\delta_e}}$ per degree	$-C_{m_{\alpha}}$ per degree	$-C_{m_{\delta_e}}$ per degree	$-(C_{m_q} + C_{m_{\dot{\alpha}}})$ per radian
0.7	0.043 ± 0.009	0.0176 ± 0.0035	0.0024 ± 0.0005	0.0079 ± 0.0016	2.80 ± 0.56
0.8	0.045 ± 0.009	0.0187 ± 0.0037	0.0030 ± 0.0006	0.0081 ± 0.0016	2.82 ± 0.56
0.9	0.049 ± 0.010	0.0199 ± 0.0040	0.0040 ± 0.0008	0.0096 ± 0.0019	3.27 ± 0.65
0.92	0.050 ± 0.010	0.0200 ± 0.0040	0.0043 ± 0.0008	0.0098 ± 0.0020	2.86 ± 0.57
0.94	0.052 ± 0.010	0.0202 ± 0.0040	0.0052 ± 0.0010	0.0101 ± 0.0020	2.45 ± 0.49
0.96	0.054 ± 0.011	0.0200 ± 0.0040	0.0061 ± 0.0012	0.0103 ± 0.0021	2.00 ± 0.40
0.98	0.056 ± 0.011	0.0193 ± 0.0039	0.0071 ± 0.0014	0.0103 ± 0.0021	1.53 ± 0.31
1.05	0.055 ± 0.011	0.0165 ± 0.0033	0.0105 ± 0.0021	0.0099 ± 0.0020	1.98 ± 0.40
1.10	0.055 ± 0.011	0.0145 ± 0.0029	0.0104 ± 0.0021	0.0090 ± 0.0018	2.02 ± 0.40
1.20	0.052 ± 0.011	0.0115 ± 0.0023	0.0093 ± 0.0019	0.0071 ± 0.0014	1.88 ± 0.38

The static pitching moment derivatives in Table 2 are based on a centre of gravity reference position at 52% of centre line chord. For each test point these values were adjusted to the correct centre of gravity position which varied gradually as fuel was used from about 51.3% of centre line chord at the lowest Mach number to about 49.2% at the highest.

Of the other parameters listed in Equation (8) the *a priori* values of G/τ and $-1/\tau$ were set to 10 and -10 respectively while all the other parameters being estimated were set to zero initially.

The *a priori* weighting assigned to each parameter was set by assuming a standard deviation, σ , equal to 20% of the parameter value. These are also indicated in Table 2. The relevant weighting was then calculated as $1/\sigma^2$. For those parameters initially set to zero a very large standard deviation was assumed leading to an effectively zero *a priori* weighting. For those parameters which have been assigned a non-zero *a priori* weighting, a comparison can be made with the "Cramer-Rao" bound which is calculated by the modified Newton-Raphson program.² A

"Cramer-Rao" bound significantly less than the *a priori* standard deviation leads to increased confidence in the extracted value of the corresponding parameter.

Finally, it is necessary to assign a weighting to each of the time histories being matched. This reflects the measurement noise in the respective channels. In the present case no differentiation was made between measurements of the pitch rate, \dot{q} , and normal acceleration, g_{nz}/V_e , channels and, following Reference 1, equal weight based on a standard deviation of 5% of the maximum value of each measured record, was assigned to both. For records not being matched (e.g. α , θ) effectively zero weighting was assigned.

4. RESULTS

In this section the results of the analysis are presented. The analysis procedure is first summarised, followed by a broad look at some general aspects related to successful parameter estimation. A detailed discussion of the results concludes this section.

4.1 Analysis Procedure

As indicated in previous sections, pitch acceleration, \ddot{q} , is required for corrections to normal accelerometer readings (Equation (9)) and angle of incidence, α , is required for modelling the non-linear pitching moment and Z-force curves (Equations (3) and (4)). Because of the lack of measured data for these two quantities a two-stage analysis process was devised. In the first stage a linear model was used and \ddot{q} corrections to normal acceleration measurements were neglected. The resulting match against measured time histories of pitch rate and normal acceleration, while not as good as ultimately obtained, produced calculated values for \ddot{q} and α . These were then used to make the required correction to the normal acceleration and to set up the full non-linear model (Equation (6)). The time history match was then repeated to complete the second stage. If necessary, a further iteration could be made based on the calculated results of the second stage, and so on. However, in the present case the two-stage process was found to produce a sufficiently good match.

4.2 General Aspects

In Reference 10 a number of conclusions were reached relating to data requirements for successful identification. These included criteria for control input frequency, data sampling rates and record length. The present data can be examined in the light of those criteria and provide some experimental validation of them. Record length, T , in all cases satisfied the requirement $\omega_n T > 14$, i.e. about two cycles of oscillation or more. The criterion on sampling rate suggested in Reference 10 was $\omega_n \Delta t < 0.14$ and, based on the natural frequencies, ω_n , shown in Figure 2 and the sampling rate of 40 per second, this requirement was not met in some cases at the higher values of ω_n . The worst cases were, as is to be expected, at the higher Mach numbers and/or lower altitude. The results of Reference 10 further indicated that fit error deteriorated fairly gradually as $\omega_n \Delta t$ increased so that some relaxation of the sampling rate criterion can be argued provided that other requirements are satisfied. A third factor, related to control input application, is the ratio of the applied doublet frequency, ω_t , to the natural frequency, ω_n . Reference 10 found that fit error deteriorated rapidly for small ω_t/ω_n and below a certain value, successful identification was not possible. For a doublet type input this minimum ratio was approximately 0.4. In the present series of tests the pilot had difficulty applying a sufficiently rapid input in some cases due to the high control forces involved. This was particularly true for the low altitude, high Mach number cases where problems also arose with the sampling rate criterion. Examination of the matched results showed that an inadequate match could be expected for ω_t/ω_n below 0.4 to 0.45 even when $\omega_n \Delta t$ was less than 0.14. On the other hand for larger values of ω_t/ω_n a satisfactory match could be obtained with $\omega_n \Delta t$ as high as 0.18. Because of these observations, test results where ω_t/ω_n was less than 0.4 have been rejected. On the other hand the maximum value for $\omega_n \Delta t$ was set at 0.18 provided that ω_t/ω_n was greater than 0.45. Application of these criteria meant that all results at 10,000 ft above $M = 0.94$ were rejected and some high Mach number results at 30,000 ft. were also rejected. A higher sampling rate may have saved some of these points but the main restriction was the difficulty encountered in applying a sufficiently rapid control input.

Another important aspect bearing on the accuracy of the results is the maintenance of constant Mach number during the course of a particular test case. This is particularly important in the transonic regime where rapid aerodynamic changes take place. Because of this, a careful examination of the measured time history of indicated airspeed (IAS) for each test was made and, for Mach number between 0.90 and 1.05, if IAS changed by more than 1% in the course of the test, the results were rejected. This led to the rejection of a further three sets of results. An exception was made for a series of doublet input manoeuvres at M approximately equal to 0.95. The results were found to be particularly sensitive to speed at this point. A slight variation of Mach number from one manoeuvre to the next produced a large variation in the results, thus indicating the rapidity with which some derivatives, in particular the pitch damping, are changing. This will be discussed further in Section 4.4.

4.3 Classification of Results

The results remaining after application of the criteria of Section 4.2 are summarised in Table 3 which presents, in columns, the true Mach number (M_T) together with the non-dimensional derivatives and "Cramer-Rao" bounds obtained for each successfully identified case. Both 10,000 ft. and 30,000 ft. results are included. The identifying numbers at the top of each column correspond to the numbers appearing in Figures 1 and 2. Numbers beginning with 1 or 3 refer to tests at 30,000 ft while those beginning with 2 are at 10,000 ft.

TABLE 3
Summary of Results

Case Item	11	12	13	14	15	16	17A	17B	18	19	110	21	22
M_T	0.69	0.79	0.89	0.91	0.935	0.935	0.95	0.95	0.97	1.04	1.08	0.70	0.79
$-C_{z_a}$ per degree	0.041 ± 0.006	0.041 ± 0.007	0.046 ± 0.007	0.048 ± 0.007	0.051 ± 0.008	0.050 ± 0.007	0.048 ± 0.009	0.044 ± 0.007	0.048 ± 0.008	0.047 ± 0.008	0.046 ± 0.008	0.039 ± 0.006	0.043 ± 0.006
$-C_{z_b}$ per degree	0.0170 ± 0.0035	0.0179 ± 0.0037	0.0198 ± 0.0040	0.0207 ± 0.0040	0.0195 ± 0.0040	0.0194 ± 0.0040	0.0195 ± 0.0039	0.0203 ± 0.0039	0.0187 ± 0.0038	0.0150 ± 0.0032	0.0127 ± 0.0028	0.0171 ± 0.0035	0.0178 ± 0.0037
$-C_{m_a}$ per degree	0.0023 ± 0.0005	0.0028 ± 0.0005	0.0038 ± 0.0007	0.0039 ± 0.0007	0.0053 ± 0.0009	0.0049 ± 0.0007	0.0086 ± 0.0013	0.0083 ± 0.0008	0.0094 ± 0.0014	0.0107 ± 0.0014	0.0099 ± 0.0014	0.0023 ± 0.0004	0.0033 ± 0.0005
$-C_{m_b}$ per degree	0.0071 ± 0.0011	0.0078 ± 0.0011	0.0101 ± 0.0014	0.0103 ± 0.0014	0.0112 ± 0.0014	0.0106 ± 0.0013	0.0091 ± 0.0015	0.0089 ± 0.0015	0.0096 ± 0.0016	0.0087 ± 0.0014	0.0075 ± 0.0013	0.0067 ± 0.0011	0.0081 ± 0.0011
$-(C_{m_q} + C_{m_i})$ per radian	1.60 ± 0.37	1.73 ± 0.39	2.24 ± 0.48	2.42 ± 0.45	2.51 ± 0.41	2.41 ± 0.39	1.75 ± 0.36	0.90 ± 0.32	1.40 ± 0.29	1.56 ± 0.35	1.51 ± 0.36	1.48 ± 0.38	1.46 ± 0.38

TABLE 3—continued

Case Item	24	31	31A	32	32A	33	34	34A	35A	35B	35C	36A	36B
M_T	0.93	0.69	0.69	0.80	0.80	0.89	0.92	0.92	0.932	0.932	0.932	0.951	0.951
C_{z_a} per degree	0.051 ± 0.007	0.040 ± 0.006	0.040 ± 0.007	0.041 ± 0.007	0.041 ± 0.006	0.043 ± 0.007	0.044 ± 0.007	0.047 ± 0.007	0.052 ± 0.008	0.054 ± 0.008	0.052 ± 0.008	0.052 ± 0.008	0.048 ± 0.007
$-C_{z_{\delta_c}}$ per degree	0.0186 ± 0.0039	0.0171 ± 0.0035	0.0174 ± 0.0035	0.0183 ± 0.0037	0.0178 ± 0.0037	0.0203 ± 0.0040	0.0199 ± 0.0039	0.0195 ± 0.0039	0.0204 ± 0.0040	0.0204 ± 0.0041	0.0200 ± 0.0040	0.0194 ± 0.0040	0.0213 ± 0.0040
$-C_{m_z}$ per degree	0.0040 ± 0.0007	0.0022 ± 0.0005	0.0021 ± 0.0004	0.0026 ± 0.0004	0.0025 ± 0.0004	0.0034 ± 0.0005	0.0039 ± 0.0006	0.0038 ± 0.0006	0.0048 ± 0.0007	0.0055 ± 0.0011	0.0048 ± 0.0007	0.0070 ± 0.0008	0.0076 ± 0.0006
$-C_{m_{\delta_c}}$ per degree	0.0097 ± 0.0014	0.0070 ± 0.0012	0.0068 ± 0.0011	0.0077 ± 0.0011	0.0071 ± 0.0011	0.0089 ± 0.0014	0.0100 ± 0.0014	0.0098 ± 0.0014	0.0109 ± 0.0014	0.0117 ± 0.0016	0.0112 ± 0.0015	0.0106 ± 0.0014	0.0105 ± 0.0015
$-(C_{m_{\delta}} + C_{m_z})$ per radian	2.41 ± 0.50	1.70 ± 0.40	1.72 ± 0.40	1.81 ± 0.40	1.63 ± 0.39	1.96 ± 0.46	2.18 ± 0.45	2.33 ± 0.45	2.48 ± 0.42	2.20 ± 0.37	2.61 ± 0.47	1.86 ± 0.35	1.36 ± 0.32

TABLE 3—Continued

Case Item	36C	37A	37B	37C	38A	39A	39B	310A	310B	311A	311B	312A	312B
M_T	0.951	0.964	0.964	0.964	1.034	1.089	1.089	1.148	1.148	1.216	1.216	1.216	1.216
$-C_{z_a}$ per degree	0.044 ± 0.007	0.047 ± 0.007	0.048 ± 0.008	0.047 ± 0.008	0.044 ± 0.007	0.043 ± 0.008	0.044 ± 0.008	0.041 ± 0.007	0.042 ± 0.007	0.041 ± 0.007	0.042 ± 0.007	0.040 ± 0.007	0.043 ± 0.007
$-C_{z_r}$ per degree	0.0212 ± 0.0040	0.0191 ± 0.0039	0.0192 ± 0.0039	0.0186 ± 0.0039	0.0158 ± 0.0033	0.0138 ± 0.0029	0.0145 ± 0.0029	0.0129 ± 0.0027	0.0126 ± 0.0027	0.0108 ± 0.0022	0.0111 ± 0.0022	0.0102 ± 0.0022	0.0105 ± 0.0022
$-C_{m_a}$ per degree	0.0087 ± 0.0011	0.0092 ± 0.0007	0.0085 ± 0.0008	0.0087 ± 0.0009	0.0097 ± 0.0013	0.0091 ± 0.0013	0.0098 ± 0.0011	0.0093 ± 0.0010	0.0093 ± 0.0010	0.0093 ± 0.0014	0.0085 ± 0.0008	0.0087 ± 0.0009	0.0090 ± 0.0009
$-C_{m_r}$ per degree	0.0107 ± 0.0017	0.0098 ± 0.0014	0.0095 ± 0.0014	0.0091 ± 0.0014	0.0087 ± 0.0013	0.0075 ± 0.0012	0.0075 ± 0.0013	0.0066 ± 0.0010	0.0064 ± 0.0010	0.0058 ± 0.0010	0.0056 ± 0.0010	0.0052 ± 0.0008	0.0055 ± 0.0009
$-(C_{m_q} + C_{m_a})$ per radian	0.89 ± 0.30	1.46 ± 0.32	1.57 ± 0.33	1.46 ± 0.33	1.50 ± 0.33	1.33 ± 0.34	1.49 ± 0.35	1.37 ± 0.31	1.39 ± 0.33	1.60 ± 0.34	1.44 ± 0.32	1.26 ± 0.30	1.52 ± 0.34

Two examples of matched results obtained are shown in Figures 3 and 4. Figure 3 corresponds to case 24 of Table 3, at $M_T = 0.93$ and 10,000 ft. This case represents a marginally acceptable match with the ratio ω_t/ω_n being low at about 0.45 and $\omega_n\Delta t$ being 0.13. The resolution of the transducer measuring elevator angle was not as good as may be generally expected as can be seen in Figure 3. However, reasonable results were obtained without recourse to further processing. Figure 4 shows the time history matches for case 310A at $M = 1.148$ and 30,000 ft. Because a full doublet input was not obtained in this case it is difficult to calculate a value for ω_t/ω_n . However, it may be estimated to be around 0.5 to 0.6, a quite adequate value. The value for $\omega_n\Delta t$ is somewhat high at about 0.17 but, nevertheless, an acceptable match was obtained for this case.

Figures 5 to 9 are plots of the results summarised in Table 3. In addition, the *a priori* values of each of the non-dimensional derivatives have been plotted in these figures and, for reference, faired estimates, obtained using the Digital Datcom Program¹¹ are also shown in Figures 5, 7 and 8.

4.4 Detailed Discussion

4.4.1 Cramer-Rao Bounds

In order to assess the level of confidence to be placed in the values of the respective derivatives extracted it is helpful to examine the corresponding "Cramer-Rao" bounds. Significant reduction of these bounds below the comparable *a priori* weightings indicates the presence of relevant information in the measured records and implies a reasonable level of confidence in the extracted value. On this basis it is clear from a comparison between the bounds shown in Table 3 and the *a priori* weightings summarised in Table 2 that a fair amount of confidence can be placed in the pitching moment derivatives (Figs. 5 to 7) including the pitch damping. Of the Z-force derivatives, only the $C_{z\alpha}$ results (Fig. 8) can be accepted with some confidence while the results for $C_{z\beta}$ indicate very little information for this derivative. Consequently its extracted values do not vary significantly from the *a priori* settings (Fig. 9). It should be noted that introduction of a larger number of parameters when using the full non-linear system of Equation (6) caused some increase in the "Cramer-Rao" bounds as compared to those obtained using the simpler linear model. This was particularly true of the pitch stiffness $C_{m\alpha}$ which is most affected by the non-linear modelling. "Cramer-Rao" bounds shown in Table 3 for $C_{m\alpha}$ are up to twice as large as those produced by the first stage linear model, but are still considerably smaller than the *a priori* weights.

4.4.2 Pitch Damping

Figure 5 summarises all the results obtained for pitch damping at both 10,000 ft and 30,000 ft. The 10,000 ft points obtained at $M_T = 0.7$ and $M_T = 0.79$ lie slightly below the 30,000 ft data indicating a possible non-linearity with incidence. However, the results lie within the 95% confidence interval obtained by application of Student's *t*-test to the results at each Mach number. The 95% confidence intervals have been evaluated wherever there are three or more data points at approximately the same Mach number, and are indicated by the bars in Figure 5. In general the scatter of the results can be regarded as good with the relatively large scatter at the highest Mach number attributable to the fact that both ω_t/ω_n and $\omega_n\Delta t$ are marginal there. The results at $M = 0.95$ are exceptional due to the apparently large scatter. However, these results, which were obtained from successive doublet inputs at a single test point reflect slight variations in airspeed from one input to the next and indicate the very rapid aerodynamic changes taking place at this Mach number.

In general, the results show extracted pitch damping values to be quite different from the *a priori* expectation. At subsonic Mach numbers the values are little more than half the *a priori* values, and rise gradually to a peak at about $M_T = 0.94$. This is followed by a rapid drop at around $M_T = 0.95$ and a recovery to reach the supersonic value at about $M_T = 0.96$ to 0.97. The supersonic value is also lower, by about 20%, than the *a priori* value. For comparison, the Digital Datcom results appear to be quite close to the extracted values at subsonic and supersonic speeds but the variation at transonic Mach numbers is quite different.

4.4.3 Other Derivatives

Pitching moment due to elevator deflection, $C_{m_{\delta_e}}$, is shown in Figure 6. Although the extracted value at $M = 0.7$ is about 15% lower than the *a priori* value, the general agreement at subsonic speeds is reasonably good. However, for Mach numbers above 1 the flight values are consistently about 20% below the *a priori* (wind tunnel) values. The 10,000 ft result appears to be somewhat lower than the 30,000 ft results at $M = 0.93$ but at the lower Mach numbers differences between 10,000 ft and 30,000 ft do not appear to be significant.

Results for pitching moment due to incidence, C_{m_α} , are summarised in Figure 7. At subsonic Mach numbers the low flight values reflect, to some extent, the non-linear pitching moment curves (see Reference 1 for a more detailed discussion). The linear model, at stage 1 calculations, generally produced significantly higher values of C_{m_α} at subsonic speeds and the ratio of the two can be seen in Figure 10. Figure 10 also shows that for Mach number greater than about 1 both the linear and non-linear models produce substantially the same value for C_{m_α} , as would be expected for a linear variation of pitching moment with incidence. Returning to Figure 7, the rise of C_{m_α} at transonic M appears to be steeper than expected from the wind tunnel results. This is consistent with the similarly rapid variation of pitch damping (Fig. 5) at about $M_T = 0.95$. At supersonic speeds, Figure 7 shows the extracted flight values to be consistently about 20% below the tunnel results, as was also noted in Figure 6 for $C_{m_{\delta_e}}$. The Digital Datcom curve shown in Figure 7 is higher than either flight or wind tunnel values throughout the Mach number range.

The variation of Z-force with incidence is shown in Figure 8. The subsonic values of C_{z_α} measured in flight appear to be somewhat lower than the tunnel values as was noted with C_{m_α} (Fig. 7). However, non-linearities are smaller in this case and the differences in C_{z_α} obtained between the linear and non-linear models are generally contained within the scatter band. The transonic flight values do not differ significantly from the tunnel values but, as with the pitching moment derivatives, the supersonic values are consistently about 20% below those obtained from tunnel data. Digital Datcom estimates lie above both tunnel and flight results throughout the Mach number range.

The fact that each of the derivatives, $C_{m_{\delta_e}}$, C_{m_α} and C_{z_α} are, at supersonic speeds, substantially less than the respective wind tunnel data suggests the possibility of an aerodynamic cause common to all. For example, the differences may be due to variations in boattail geometry between tunnel model and full scale aircraft, sting interference effects, or differences in flow development particularly about the elevators and/or wing-body and fin-body junctions due to Reynolds number differences. Further detailed investigation would be required to resolve these questions.

Finally, the results for Z-force due to elevator deflection, $C_{z_{\delta_e}}$, are summarised in Figure 9, which shows good agreement between flight and *a priori* values. However, as indicated in Section 4.4.1, the flight measurements contained very little information on this derivative and little confidence can be placed in the results extracted from flight. The *a priori* weighting in this case prevents the results from straying too far from the *a priori* values.

5. CONCLUDING REMARKS

A flight investigation has been undertaken to determine the pitch damping of a 60-degree delta wing aircraft in the Mach number range from $M = 0.7$ to $M = 1.2$. The flight tests, including instrumentation and data recording, were conducted by ARDU and the data provided to ARL for analysis. Apart from pitch damping, results were also obtained for pitching moment and Z-force derivatives with respect to incidence and elevator deflection. In addition, further information relating to the effects of sampling rate and control input on the identification process was obtained. This will have a bearing on future flight test planning.

A modified Newton-Raphson parameter estimation procedure was used to analyse flight records of normal acceleration and pitch rate obtained in response to an elevator input. The mathematical model describing the aircraft motion made provision for non-linearities in pitching moment and Z-force with incidence, and also took account of a time lag in the pitch rate measurements.

The results for pitch damping were significantly different throughout the Mach number range from the values previously assumed. The new values are up to 50% less at subsonic speeds

and about 20%, less at supersonic speeds with a rapid transition taking place at about $M = 0.95$. Of the other derivatives extracted, pitching moment changes due to incidence and elevator deflection and Z-force change due to incidence all had values about 20%, below the respective wind tunnel results at Mach numbers above about 1, while showing closer agreement at lower Mach numbers. Further investigation would be required to isolate the causes of the differences at supersonic speeds. The final derivative, for Z-force change with elevator deflection, could not be extracted with confidence due to insufficient information contained in the measurements.

In summary, the parameter estimation procedure has proved to be an effective analysis method even when only a modest amount of flight data, obtained using a relatively simple instrumentation package, has been available. Most derivatives have been successfully extracted with relatively small amounts of scatter and trends have been clearly identified.

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NOTATION

A	Matrix of stability parameters, Equation (1)
b	Measurement bias vector
B	Matrix of control parameters, Equation (1)
c	Unknown parameter vector
\bar{c}	Mean aerodynamic chord
C_m	Non-dimensional pitching moment coefficient, $M/(\frac{1}{2}\rho V_e^2 \delta \bar{c})$
C_z	Non-dimensional Z-force coefficient, $Z/(\frac{1}{2}\rho V_e^2 S)$
C_{m_q}	Non-dimensional moment derivative w.r.t. q , $\partial C_m / \partial (q \bar{c} / 2 V_e)$
C_{m_α}	Non-dimensional moment derivative w.r.t. α , $\partial C_m / \partial \alpha$
$C_{m_{\dot{\alpha}}}$	Non-dimensional moment derivative w.r.t. α , $\partial C_m / \partial (\alpha \bar{c} / 2 V_e)$
$C_{m_{\delta_e}}$	Non-dimensional moment derivative w.r.t. δ_e , $\partial C_m / \partial \delta_e$
C_{z_α}	Non-dimensional Z-force derivative w.r.t. α , $\partial C_z / \partial \alpha$
$C_{z_{\delta_e}}$	Non-dimensional Z-force derivative w.r.t. δ_e , $\partial C_z / \partial \delta_e$
F	Matrix of stability parameters, Equation (2)
g	Gravitational acceleration
G	Matrix of control parameters, Equation (2), or gain, Equation (5)
I_y	Moment of inertia in pitch, kg-m ²
m	Mass, kg
M	Mach number or pitching moment
M_0	Equation error bias term, Equation (6)
M_q	Pitching moment derivative w.r.t. q
M_α	Pitching moment derivative w.r.t. α
M_{α^2}	Pitching moment derivative w.r.t. α^2
M_{α^3}	Pitching moment derivative w.r.t. α^3
$M_{\dot{\alpha}}$	Pitching moment derivative w.r.t. α
M_{δ_e}	Pitching moment derivative w.r.t. δ_e
n_z	Normal acceleration in g units
q	Pitch rate, rad/s
S	Reference wing area, m ²
t	Time, sec
T	Length of measured responses, sec
u	Vector of control input variables
V	Resultant airspeed, knots

\mathbf{x}	Vector of state variables
x	Body axis co-ordinate in forward direction
\mathbf{y}	Calculated response vector, Equation (2)
z	Body axis co-ordinate in down direction
Z	Force in z-direction, N
Z_0	Equation error bias term, Equation (6)
Z_q	Z-force derivative w.r.t. q
Z_α	Z-force derivative w.r.t. α
Z_{α^2}	Z-force derivative w.r.t. α^2
Z_{α^3}	Z-force derivative w.r.t. α^3
Z_{δ_e}	Z-force derivative w.r.t. δ_e
α	Angle of incidence increment, rad
Δ	Increment
δ_e	Elevator angle increment, positive t.e. down, rad
ζ	Damping ratio
θ	Pitch attitude increment
ρ	Air density, kg/m ³
σ	Standard deviation
τ	Time constant, sec
ω_f	Doublet forcing frequency, rad/s
ω_n	Natural undamped frequency, rad/s

Subscripts

a	Normal accelerometer
e	Trim or equilibrium state
m	Measured value
T	True value

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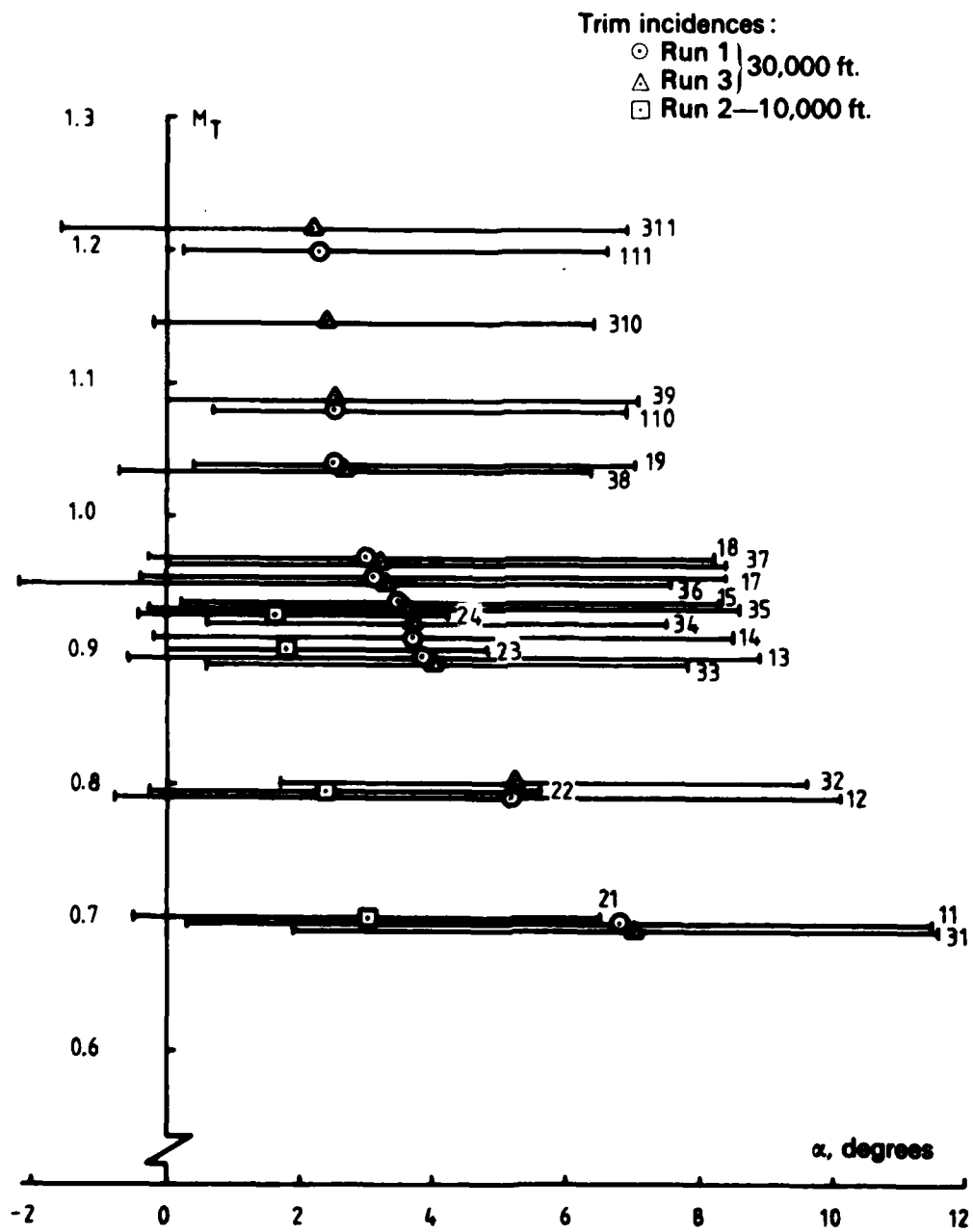


FIG. 1: INCIDENCE RANGE OF TESTS—SUMMARY

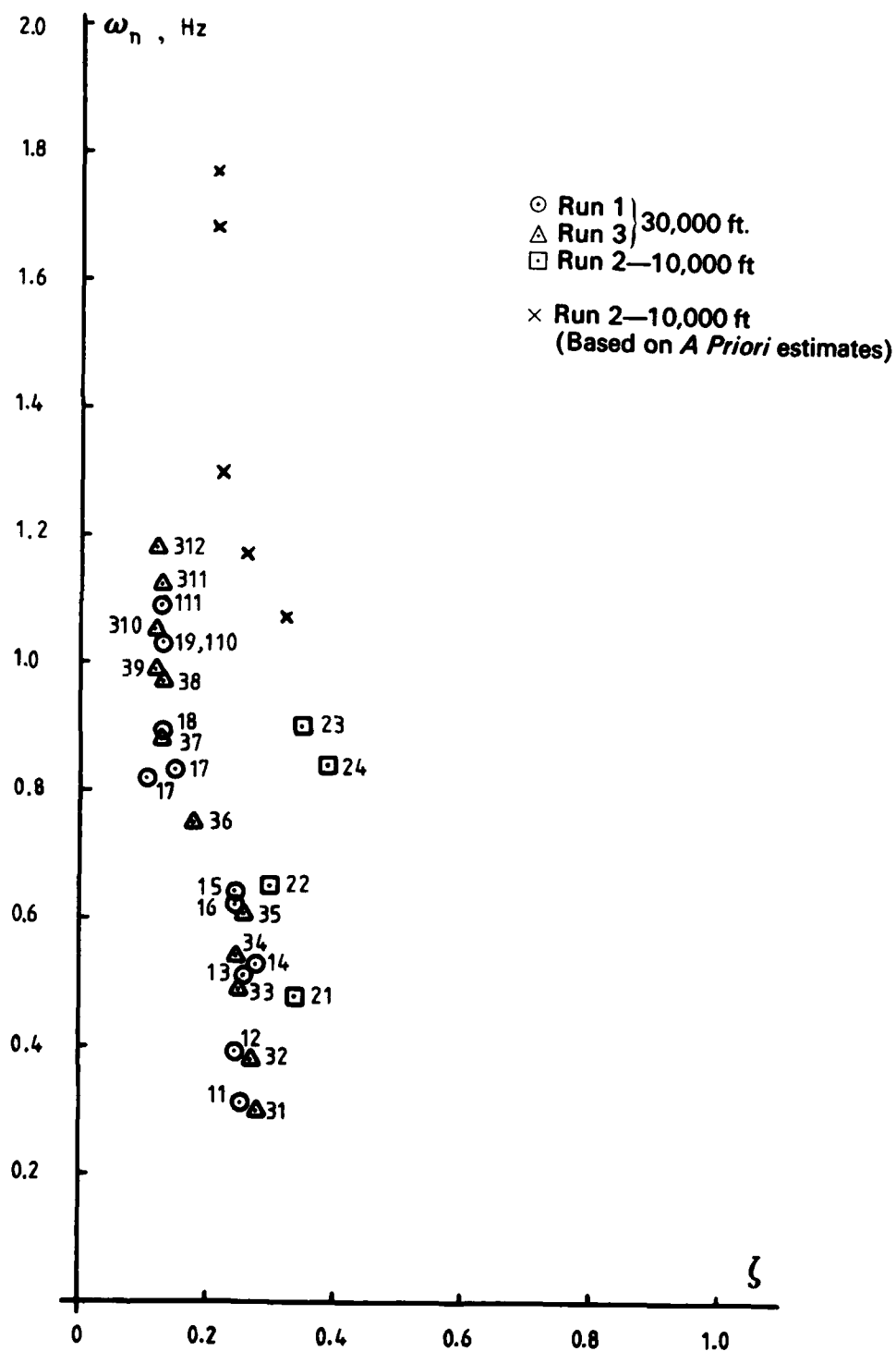


FIG. 2: FREQUENCY AND DAMPING OF SHORT PERIOD—SUMMARY

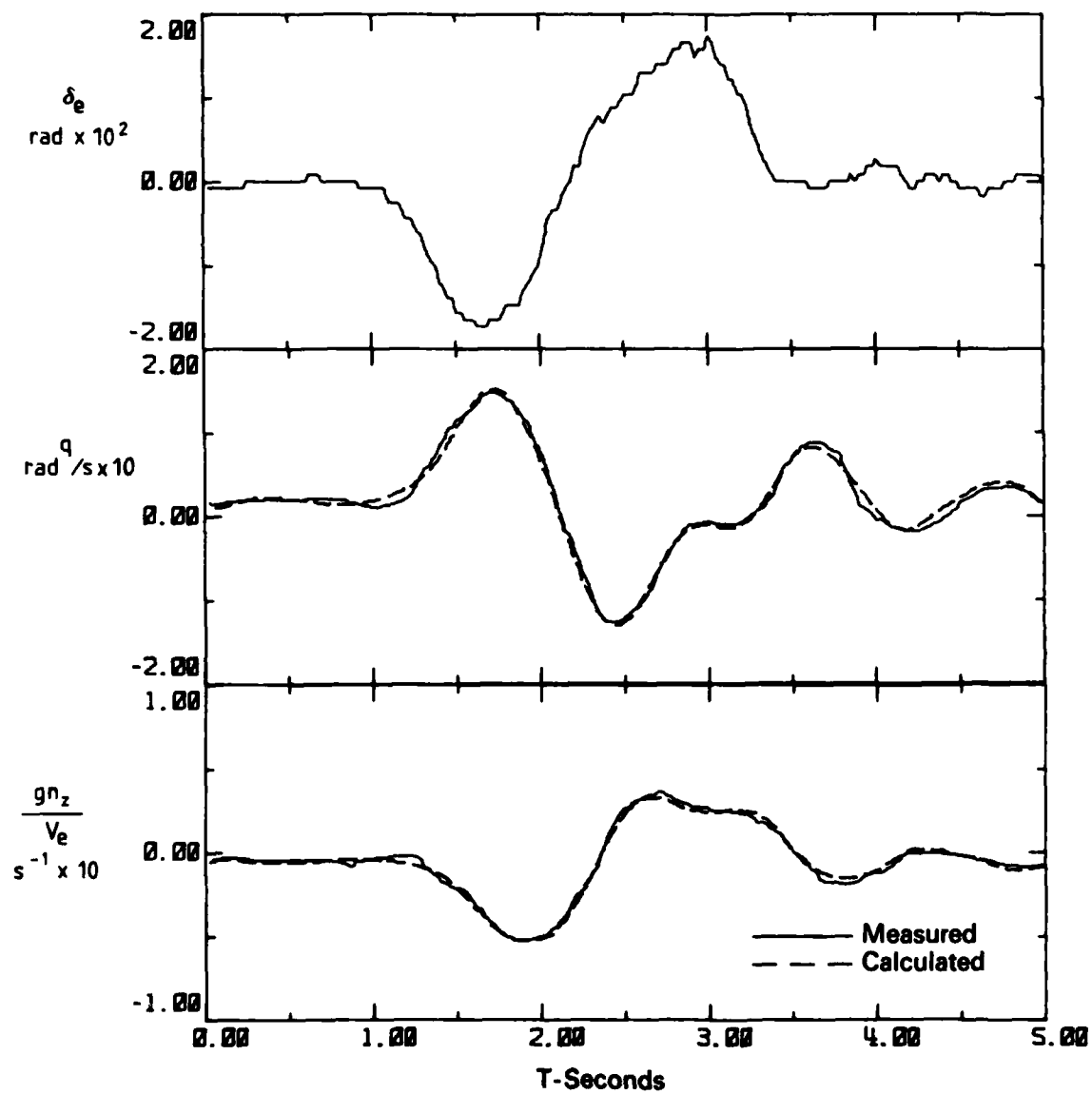


FIG. 3: MATCHED RESULTS—CASE 24, $M = 0.93$, 10,000 ft.

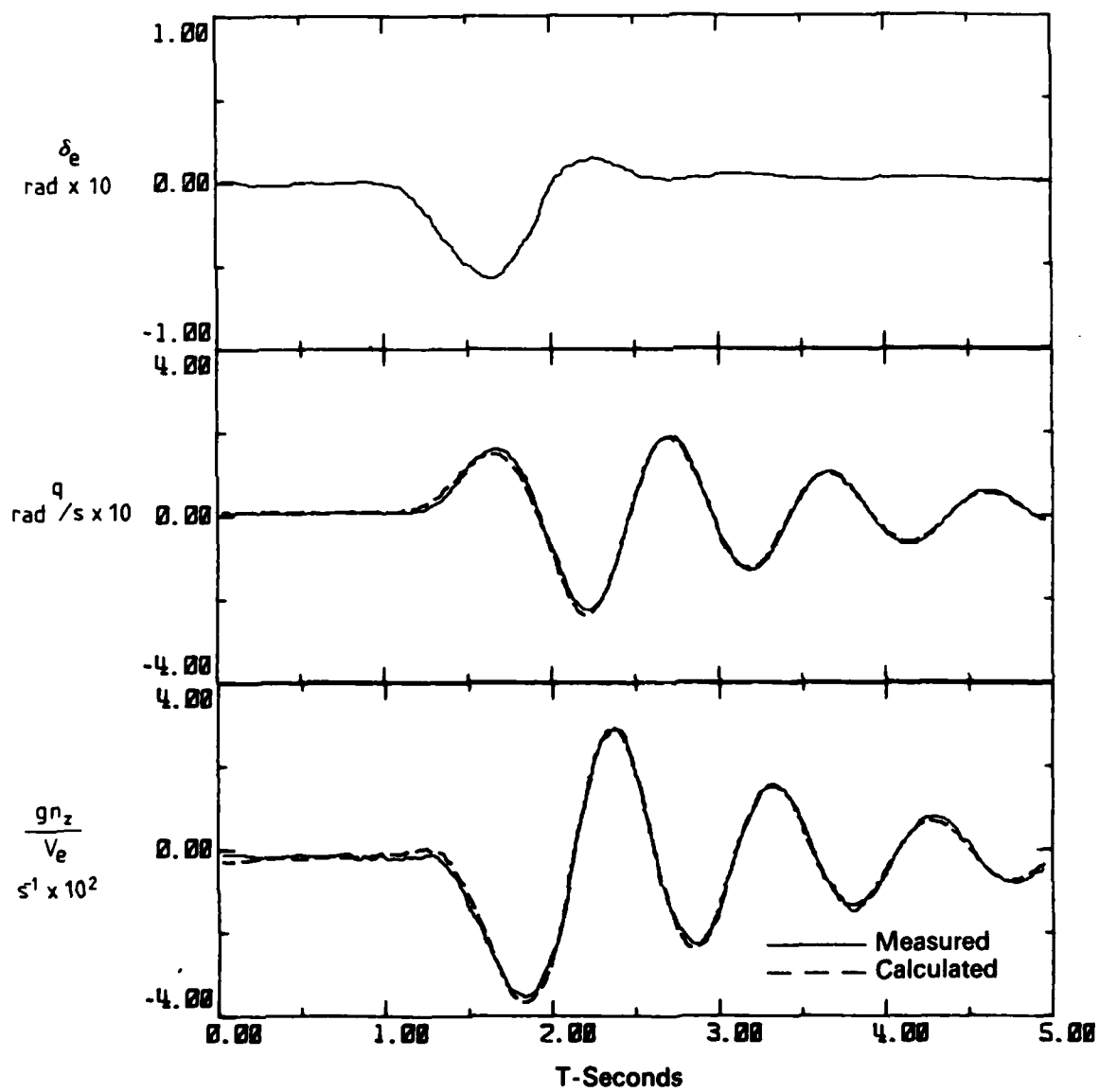


FIG. 4: MATCHED RESULTS—CASE 310A, $M = 1.148$, 30,000 ft.

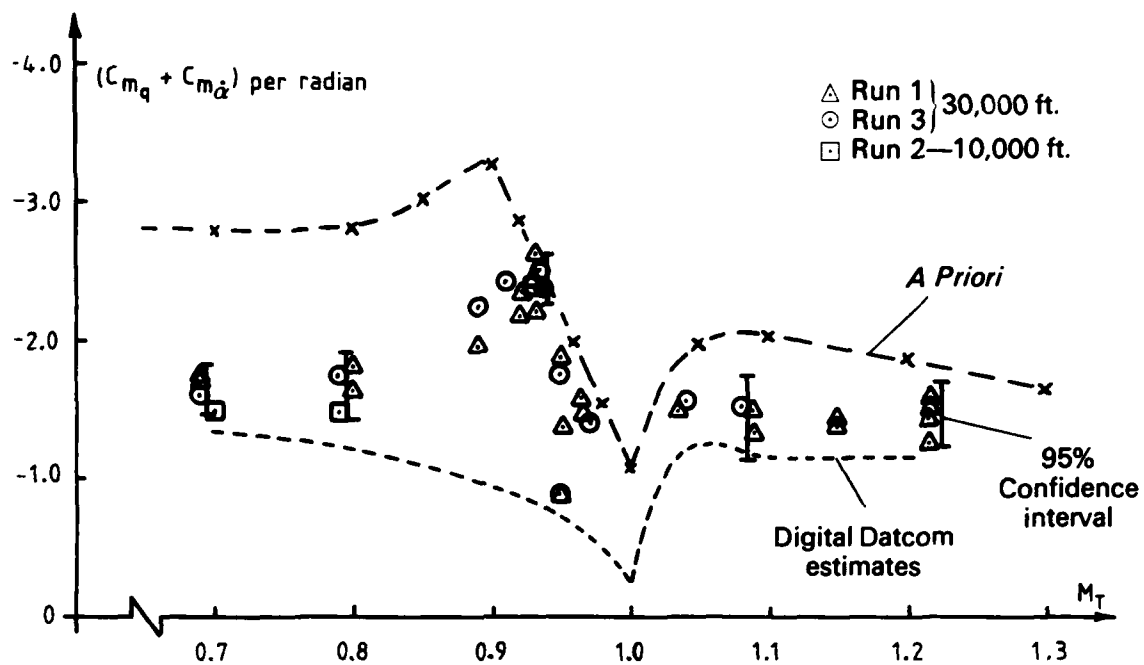


FIG. 5: VARIATION OF PITCH DAMPING WITH MACH NUMBER

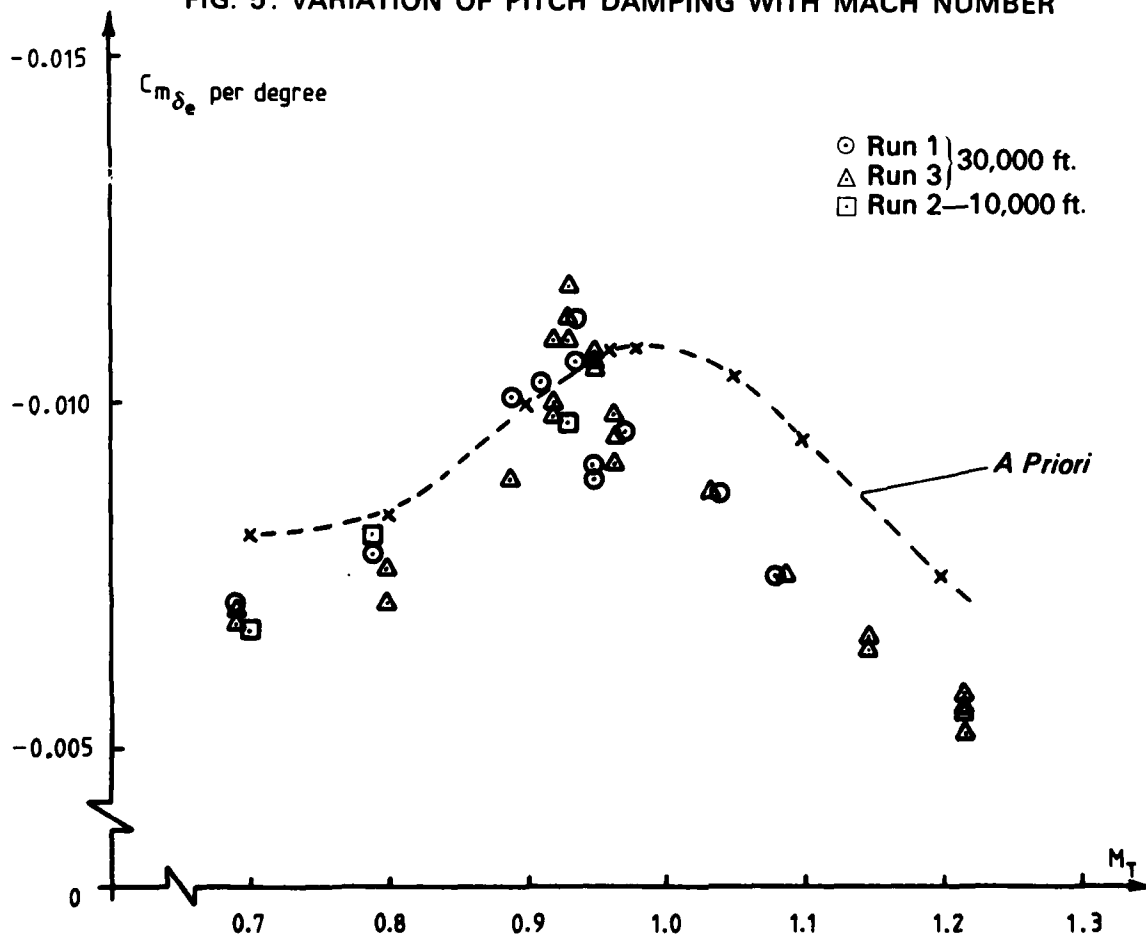


FIG. 6: PITCHING MOMENT DUE TO ELEVATOR vs MACH NUMBER

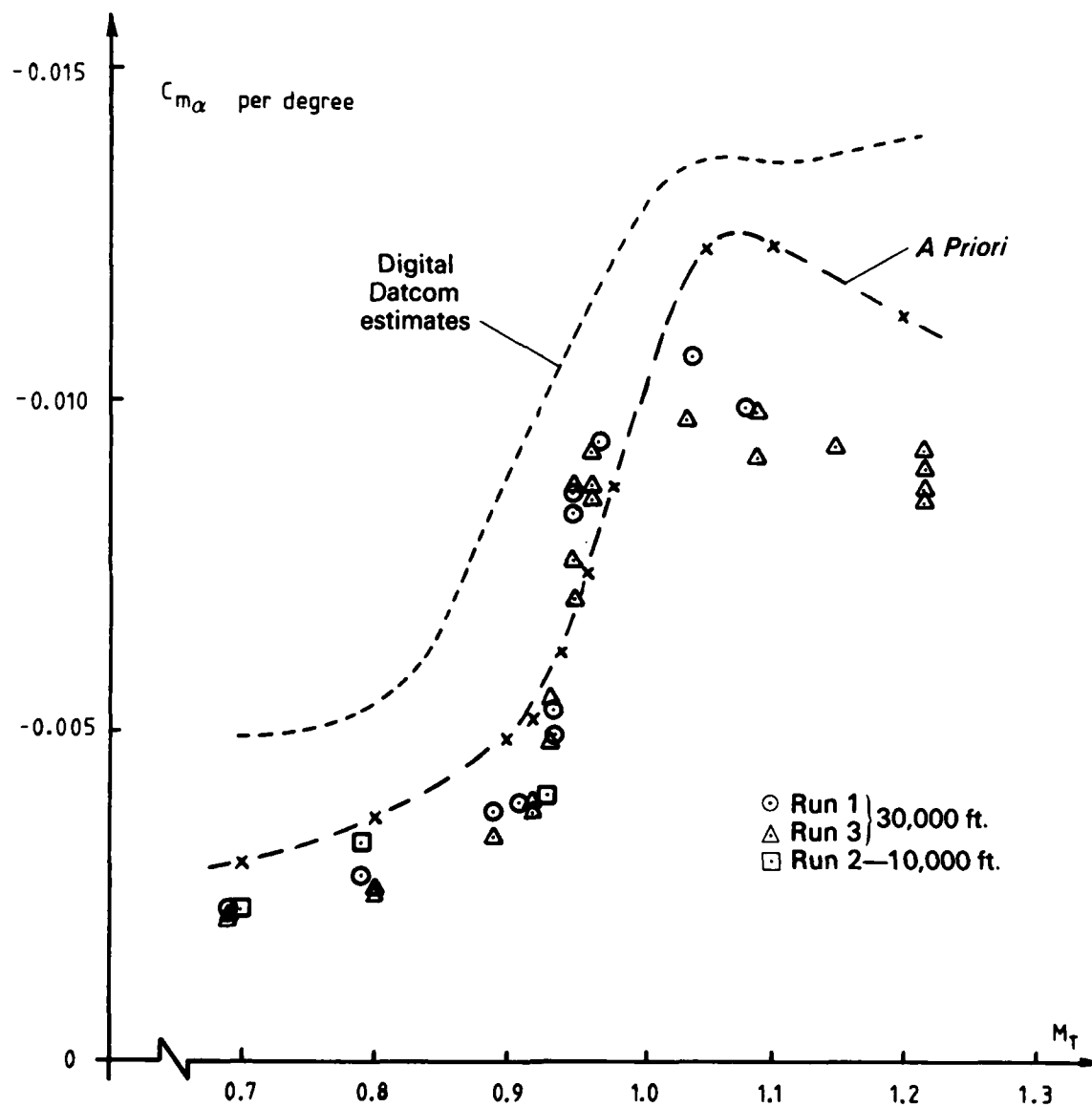


FIG. 7: PITCHING MOMENT DUE TO INCIDENCE vs MACH NUMBER

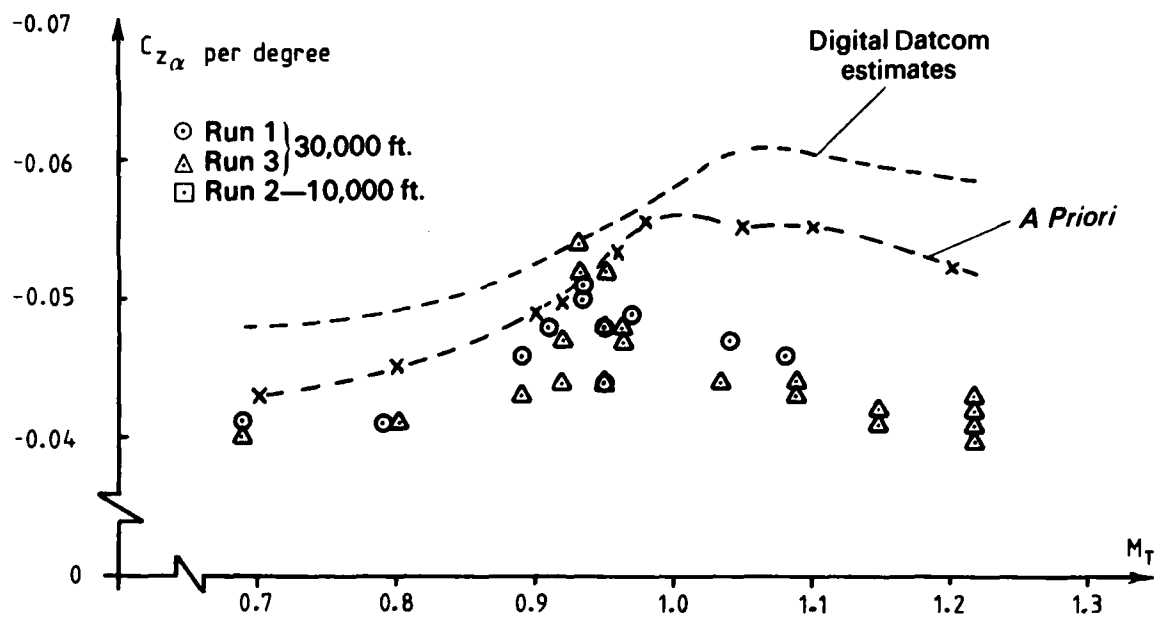


FIG. 8: Z-FORCE DUE TO INCIDENCE vs MACH NUMBER

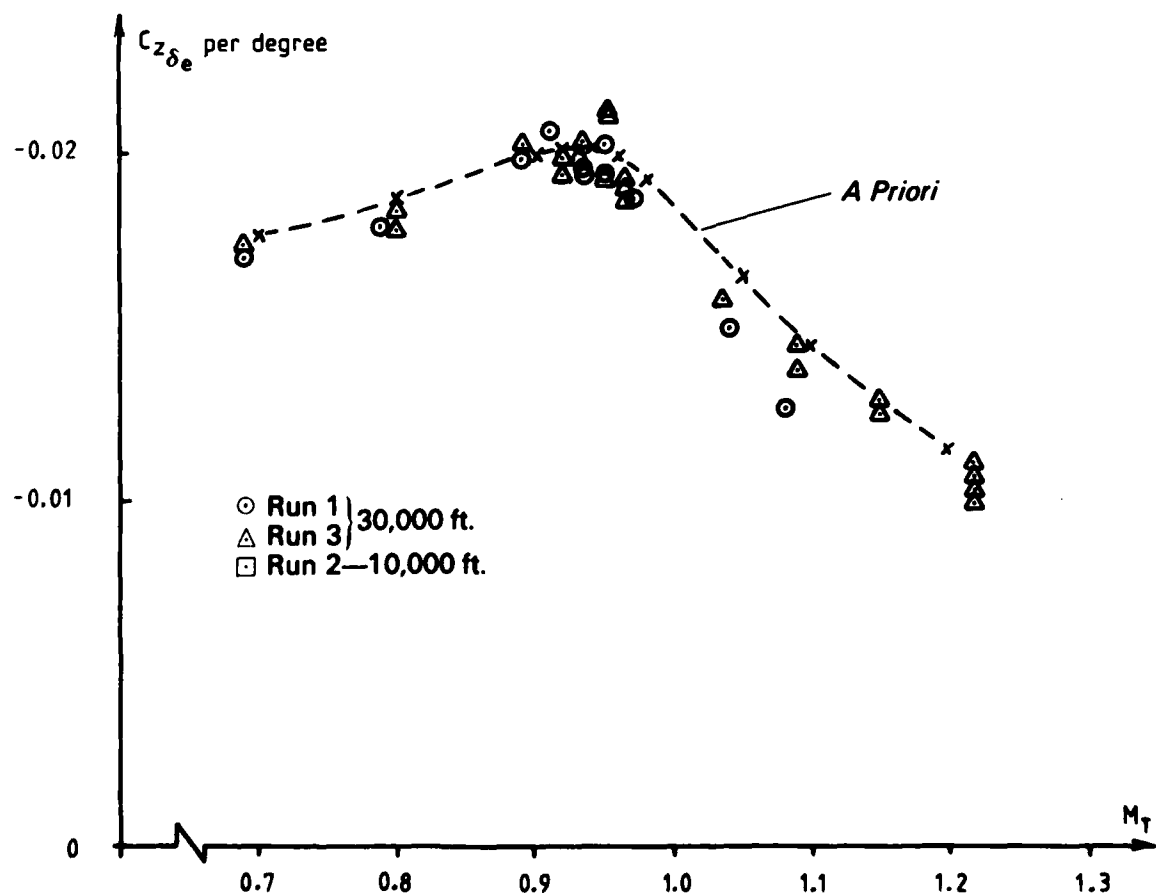


FIG. 9: Z-FORCE DUE TO ELEVATOR vs MACH NUMBER

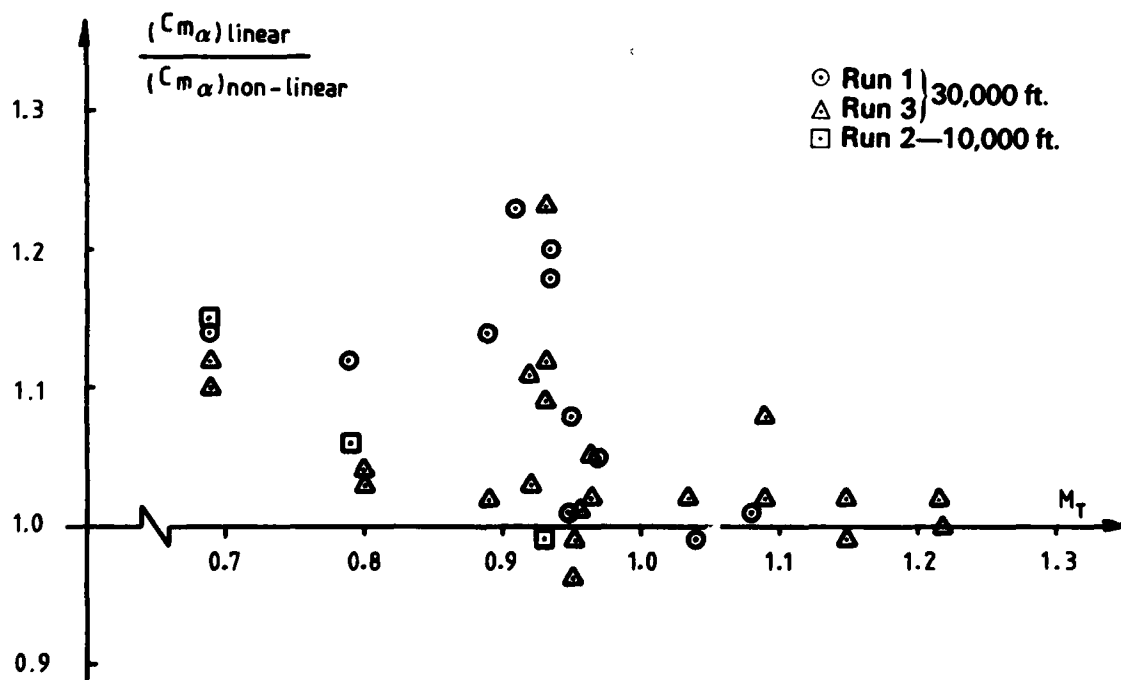


FIG. 10: RATIO OF LINEAR AND NON-LINEAR MODEL RESULTS FOR C_{m_α}

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